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Exercise 3.1 Pg 3. (Lecture Note)

9a) Let β be one root and 2β be the other root.

$$3x^2 + kx + 96 = 0$$
$$a = 3, \quad b = k, \quad c = 96$$

Sum of roots
$$\beta + 2\beta = \frac{-b}{a}$$
$$3\beta = -\frac{k}{3} \quad \cdots (1)$$

product of roots
$$\beta(2\beta) = \frac{c}{a}$$
$$2\beta^2 = \frac{96}{3}$$
$$2\beta^2 = 32$$
$$\beta^2 = \frac{32}{2}$$
$$\beta^2 = 16$$
$$\beta = \pm\sqrt{16}$$
$$\beta = \pm 4$$

$\therefore$ $\beta = 4$ (as roots are positive)

$\therefore$ one root is 4 # and the other is 8 #

Sub $\beta = 4$ into eq^n (1)

$$3\beta = -\frac{k}{3}$$
$$3(4) = \frac{-k}{3}$$
$$12 = \frac{-k}{3}$$
$$k = -36 \ \#$$

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10) $3x^2 - 3kx + k - 6 = 0$.

$a = 3$, $b = -3k$ $c = k-6$.

sum of roots

$$\alpha+\beta = -\frac{b}{a}$$

$$\alpha+\beta = -\frac{(-3k)}{3}$$

$$\alpha+\beta = k$$

product of roots

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{k-6}{3}$$

special identity.

$$\alpha^2+\beta^2 = \frac{20}{3}$$

$$(\alpha+\beta)^2 - 2\alpha\beta = \frac{20}{3}$$

$$k^2 - 2\left(\frac{k-6}{3}\right) = \frac{20}{3}$$

$\times 3$ throughout

$$3k^2 - 2(k-6) = 20$$

$$3k^2 - 2k + 12 = 20$$

$$3k^2 - 2k + 12 - 20 = 0$$

$$3k^2 - 2k - 8 = 0$$

$$(3k+4)(k-2) = 0$$

$$k = -\frac{4}{3} \text{ # } \quad \text{or} \quad k = 2 \text{ # }$$

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11a) $4x^2 - 1x + 36 = 0$

$a = 4, \quad b = -1, \quad c = 36$

Sum of roots
$$\alpha^2 + \beta^2 = -\frac{b}{a}$$
$$\alpha^2 + \beta^2 = \frac{-(-1)}{4}$$
$$\alpha^2 + \beta^2 = \frac{1}{4}$$

product of roots
$$\alpha^2\beta^2 = \frac{c}{a}$$
$$\alpha^2\beta^2 = \frac{36}{4}$$
$$\alpha^2\beta^2 = 9$$

sum of new roots
$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$
$$= \frac{\frac{1}{4}}{9}$$
$$= \frac{1}{4} \div 9 = \frac{1}{36}$$

product of NEW roots
$$\frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{\alpha^2\beta^2}$$
$$= \frac{1}{9}$$

$\therefore$ equation: $x^2 - (\text{sum})x + (\text{product}) = 0$

$$x^2 - \frac{1}{36}x + \frac{1}{9} = 0$$

$\times 36$ throughout

$$\boxed{36x^2 - x + 4 = 0}$$ #

11b) $\alpha^2\beta^2 = 9$

$(\alpha\beta)^2 = 9$

$\alpha\beta = \pm\sqrt{9}$

$\alpha\beta = \pm 3.$

$\alpha^2 + \beta^2 = \frac{1}{4}$

$(\alpha+\beta)^2 - 2\alpha\beta = \frac{1}{4}$

When $\alpha\beta = 3$,

$(\alpha+\beta)^2 - 2(3) = \frac{1}{4}$

$(\alpha+\beta)^2 - 6 = \frac{1}{4}$

$(\alpha+\beta)^2 = \frac{1}{4} + 6$

$(\alpha+\beta)^2 = 6\frac{1}{4}$

$\alpha+\beta = \pm\sqrt{6\frac{1}{4}}$

$\alpha+\beta = \pm 2\frac{1}{2}$

When $\alpha\beta = -3$,

$(\alpha+\beta)^2 - 2(-3) = \frac{1}{4}$

$(\alpha+\beta)^2 = -5\frac{3}{4}$

cannot be solved.

1st eq^{n} 2nd eq^{n}

$\boxed{\alpha\beta = 3, \ \alpha+\beta = 2\frac{1}{2}}$ $\boxed{\alpha\beta = 3, \ \alpha+\beta = -2\frac{1}{2}}$

$\therefore$ The two equations are:

$x^2 - (\text{sum})x + (\text{product}) = 0$

1st : $x^2 - \frac{5}{2}x + 3 = 0$

$2x^2 - 5x + 6 = 0$ #

2nd : $x^2 - (-\frac{5}{2})x + 3 = 0$

$x^2 + 5x + 6 = 0$ #